$Z_2 \times Z_2$ orbifold compactification as the origin of realistic free fermionic models

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ABSTRACT

I discuss the correspondence between realistic four dimensional free fermionic models and $Z_2 \times Z_2$ orbifold compactification. I discuss the properties of the $Z_2 \times Z_2$ orbifold that are reflected in the realistic free fermionic models. I argue that the properties of the realistic free fermionic models arise due to the underlying $Z_2 \times Z_2$ orbifold compactification with nontrivial background fields. I suggest that three generation is a natural outcome of $Z_2 \times Z_2$ orbifold compactification with "standard embedding" and at the point in compactification space that corresponds to the free fermionic formulation. I discuss how quark flavor mixing is related to the compactification.

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1. Introduction

Superstring theory may consistently unify gravity with the gauge interactions. The consistency of superstring theory imposes a certain number of degrees of freedom. In the closed heterotic string [1], of the 26 right–moving bosonic degrees of freedom, 16 are compactified on a flat torus and produce the observable and hidden gauge groups. Six right–moving bosonic degrees of freedom, combined with six left–moving degrees of freedom, are compactified on Calabi–Yau manifold [2] or on an orbifold [3]. Alternatively, all the extra degrees of freedom, beyond the four space–time dimensions, can be taken as bosonic [4,5], or fermionic [6], internal degrees of freedom propagating on the string world–sheet. The different interpretations are expected to be related. In this paper, I discuss the correspondence between realistic models in the free fermionic formulation and $Z_2 \times Z_2$ orbifold compactification.

Models constructed in the free fermionic formulation produced the most realistic superstring models to date [7–15]. In Ref. [13] it was shown that the reduction of the number of chiral generations to three generations is correlated in these models with the factorization of the gauge group to observable and hidden sectors and with the breaking of nonabelian horizontal gauge groups to factors of $U(1)$'s at most. It was consequently argued that three generation is the most natural number of generations in this class of models. In Refs. [10-12] it was suggested that the generation mass hierarchy and the suppression of mixing terms among these generations is explained in terms of horizontal symmetries that are derived in these models. While the fermionic formulation enables the construction of rather realistic models, the orbifold formulation may relate the realistic models to the geometry at the unification scale. An apparent example is the number of generations which is related to the Euler characteristic in the orbifold formulation. Thus, the two formulations have complementary merits. Therefore, it is important to understand the connection between the two formulations.

The paper is organized as follows: in section 2, I present an $E_6 \times U(1)^2 \times E_8 \times$

$S0(4)^3$ model in the free fermionic formulation and its matter content. In Section 3 I show how the same model is obtained in the orbifold language. In section 4 I discuss some of the properties of the realistic free fermionic models and their relation to the $Z_2 \times Z_2$ orbifold.

2. The model in the free fermionic formulation

In the free fermionic formulation of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. The world–sheet supercurrent is realized nonlinearly among the internal left–moving free fermions,

$$T_F = \psi^\mu \partial X_\mu + i \chi^I y^I \omega^I, \quad (I = 1, \cdots, 6),$$

where X^μ, ψ^μ are the usual space–time fields and indices, $\{\chi^I, y^I, \omega^I\}$ $(i = 1, \cdots, 6)$ are 18 real free fermions transforming as the adjoint representation of $SU(2)^6$. The right–moving sector consist of $\bar{X}^\mu$ and 44 real internal free fermion fields.

Under parallel transport around a noncontractible loop of the torus the fermionic states pick up a phase. The phases for all world–sheet fermions are specified in 64 dimensional boundary condition vectors for all world–sheet fermions. A model in this construction is specified by a set of boundary condition basis vectors that spans a finite additive group Ξ. These basis vectors are constrained by the string consistency requirements (e.g. modular invariance) and completely determine the vacuum structure of the model. The physical spectrum is obtained by applying the generalized GSO projections. The low energy effective field theory is obtained by S–matrix elements between external states. The Yukawa couplings and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators. For a correlator to be nonvanishing all the symmetries of the model must be conserved. Thus, the boundary condition vectors determine the phenomenology of the models.

The six basis vectors (including the vector **1**) that generate the model in the free fermionic formulation are

$$S = (\underbrace{1, \cdots, 1}_{\psi^\mu, \chi^{1,\dots,6}}, 0, \cdots, 0|0, \cdots, 0) \tag{1a}$$

$$\xi_1 = (0, \cdots, 0| \underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}}, 0, \cdots, 0) \tag{1b}$$

$$\xi_2 = (0, \cdots, 0|0, \cdots, 0, \underbrace{1, \cdots, 1}_{\bar{\phi}^{1,\dots,8}}) \tag{1c}$$

$$b_1 = (\underbrace{1, \cdots\cdots\cdots, 1}_{\psi^\mu, \chi^{12}, y^{3,\dots,6}, \bar{y}^{3,\dots,6}}, 0, \cdots, 0|\underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\dots,5}, \bar{\eta}^1}, 0, \cdots, 0) \tag{1d}$$

$$b_2 = (\underbrace{1, \cdots\cdots\cdots\cdots, 1}_{\psi^\mu, \chi^{34}, y^{1,2}, \omega^{5,6}, \bar{y}^{1,2}\bar{\omega}^{5,6}}, 0, \cdots, 0|\underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\dots,5}, \bar{\eta}^2}, 0, \cdots, 0) \tag{1e}$$

with the choice of generalized GSO projections

$$c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = c\begin{pmatrix} b_i \\ S \end{pmatrix} = c\begin{pmatrix} b_i \\ \xi_1, \xi_2 \end{pmatrix} = c\begin{pmatrix} \xi_i \\ \xi_i \end{pmatrix} = -c\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = -c\begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1, \tag{2}$$

and the others given by modular invariance. The notation of refs. [11] is used. The first four vectors in the basis $\{\mathbf{1}, S, \xi_1, \xi_2\}$ generate a model with $N = 4$ space–time supersymmetry with an $E_8 \times E_8 \times SO(12)$ gauge group. The sector **S** generates $N = 4$ space–time supersymmetry. The first and second E_8 are obtained from the world–sheet fermionic states $\{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$ and $\{\bar{\phi}^{1,\dots,8}\}$, respectively while $SO(12)$ is obtained from $\{\bar{y}, \bar{\omega}\}^{1,\dots,6}$. The Neveu–Schwarz sector produces the adjoint representations of $SO(12) \times SO(16) \times SO(16)$. The sectors ξ_1 and ξ_2 produce the spinorial representation of $SO(16)$ of the observable and hidden sectors respectively, and complete the observable and hidden gauge groups to $E_8 \times E_8$.

The vectors b_1 and b_2 break the gauge symmetry to $E_6 \times U(1)^2 \times E_8 \times SO(4)^3$ and $N = 4$ to $N = 1$ space–time supersymmetry. Restricting $b_j \cdot S = 0 \mod 2$, and $c\begin{pmatrix} S \\ b_j \end{pmatrix} = \delta_{b_j}$, for all basis vector $b_j \epsilon B$ guarantees the existence of $N = 1$ space–time supersymmetry. The superpartners from a given sector $\alpha \epsilon \Xi$ are obtained

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from the sector $S + \alpha$. We denote the $U(1)$ generators, that are generated by the world–sheet currents $: \bar{\eta}^i \bar{\eta}^{i*} :$, by $U(1)_i$. The fermionic states $\{\chi^{12}, \chi^{34}, \chi^{56}\}$ and $\{\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$ give the usual "standard– embedding", with $b(\chi^{12}, \chi^{34}, \chi^{56}) = b(\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3)$. The $U(1)$ current of the left–moving $N = 2$ world–sheet supersymmetry is given by $J(z) = i\partial_z(\chi^{12} + \chi^{34} + \chi^{56})$. The $U(1)$ charges in the decomposition of E_6 under $SO(10) \times U(1)$ are given by $U(1)_{E_6} = U(1)_1 + U(1)_2 + U(1)_3$ while the charges of the two orthogonal combinations are given by $U(1)' = U(1)_1 - U(1)_2$ and $U(1)'' = U(1)_1 + U(1)_2 - 2U(1)_3$. The three $SO(4)$ gauge groups are produced by the right–moving world–sheet fermionic states $\{\bar{y}^{3,\cdots,6}\}$, $\{\bar{y}^1, \bar{y}^2\bar{\omega}^5, \bar{\omega}^6\}$ and $\{\bar{\omega}^{1,\cdots,4}\}$.

The massless spectrum of the model consist of the following sectors. The Neveu–Schwarz and ξ_1 sectors produce in addition to the spin 2 and spin 1 states three copies of chiral multiplets that transform as $27 + \bar{27}$ under E_6, and an equal number of E_6 singlets that are charged under $U(1)^2$. The Neveu–Schwarz sector also produces three scalar multiplets that transform as $(4, \bar{4}, 1)$, one under each of the horizontal $SO(4)$ symmetries. The sectors b_1, b_2 and $b_3 = 1 + \xi_2 + b_1, b_2$ plus $b_j + \xi_2$ produce 24 chiral 27 of E_6, and 24 E_6 singlets that are charged under $U(1)_1$, $U(1)_2$. In the decomposition of E_6 under $SO(10)$, the sectors b_j produce the 16 representation of $SO(10)$ while the sectors $b_j + \xi_2$ produce the $10 + 1$ in the 27 representation of E_6. The sectors $b_j + \xi_1$ produce an equal number of E_6 singlets. The singlet of $SO(10)$ in the 27 of E_6 and the additional E_6 singlet from the sectors $b_j + \xi_1$ are produced by acting on the degenerate vacuum with $\bar{\eta}_j$ and $\bar{\eta}_j^*$. In addition to these states the sectors $b_j + \xi_1$ produce $E_6 \times E_8$ singlets which carry $U(1)^2$ charges and that transform nontrivialy under the horizontal $SO(4)$ symmetries.

In this model the only internal fermionic states which count the multiplets of E_6 are the real internal fermions $\{y, w | \bar{y}, \bar{\omega}\}$. This is observed by writing the degenerate vacuum of the sectors b_j in a combinatorial notation. The vacuum of the sectors b_j contains twelve periodic fermions. Each periodic fermion gives rise to a two dimensional degenerate vacuum $|+\rangle$ and $|-\rangle$ with fermion numbers 0 and

−1, respectively. The GSO operator, is a generalized parity, operator which selects states with definite parity. After applying the GSO projections, we can write the degenerate vacuum of the sector b_1 in combinatorial form

$$\left[\binom{4}{0}+\binom{4}{2}+\binom{4}{4}\right]\left\{\binom{2}{0}\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right]\binom{1}{0}\right.$$
$$\left.+\binom{2}{2}\left[\binom{5}{1}+\binom{5}{3}+\binom{5}{5}\right]\binom{1}{1}\right\} \qquad (3)$$

where $4 = \{y^3y^4, y^5y^6, \bar{y}^3\bar{y}^4, \bar{y}^5\bar{y}^6\}$, $2 = \{\psi^\mu, \chi^{12}\}$, $5 = \{\bar{\psi}^{1,\cdots,5}\}$ and $1 = \{\bar{\eta}^1\}$. The combinatorial factor counts the number of $|-\rangle$ in the degenerate vacuum of a given state. The two terms in the curly brackets correspond to the two components of a Weyl spinor. The $10+1$ in the 27 of E_6 are obtained from the sector $b_j + X$. From Eq. (4) it is observed that the states which count the multiplicities of E_6 are the internal fermionic states $\{y^{3,\cdots,6}|\bar{y}^{3,\cdots,6}\}$. A similar result is obtained for the sectors b_2 and b_3 with $\{y^{1,2}, \omega^{5,6}|\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\omega^{1,\cdots,4}|\bar{\omega}^{1,\cdots,4}\}$ respectively, which suggests that these twelve states correspond to a six dimensional compactified orbifold with Euler characteristic equal to 48.

3. The model in the orbifold formulation

I now describe how to construct the same model in the orbifold formulation. In the orbifold formulation [3] one starts with a model compactified on a flat torus with nontrivial background fields [5]. The action for the six compactified dimensions is given by,

$$S = \frac{1}{8\pi}\int d^2\sigma(G_{ij}\partial X^i\partial X_j + B_{ij}\partial X_i\partial X_j) \qquad (4)$$

where

$$G_{ij} = \frac{1}{2}\sum_{I=1}^{D}R_ie_i^IR_je_j^I \qquad (5)$$

is the metric of the six dimensional compactified space and $B_{ij} = -B_{ji}$ is the

antisymmetric tensor field. The $e^i = \{e_i^I\}$ are six linear independent vectors normalized to $(e_i)^2 = 2$. The left– and right–moving momenta are given by

$$P_{R,L}^I = [m_i - \frac{1}{2}(B_{ij} \pm G_{ij})n_j]e_i^{I*} \tag{6}$$

where the e_i^{I*} are dual to the e_i, and $e_i^* \cdot e_j = \delta_{ij}$. The left– and right–moving momenta span a Lorentzian even self–dual lattice. The mass formula for the left and right movers is

$$M_L^2 = -c + \frac{P_L \cdot P_L}{2} + N_L = -1 + \frac{P_R \cdot P_R}{2} + N_R = M_R^2 \tag{7}$$

where $N_{L,R}$ are the sum on the left–moving and right–moving oscillators and c is a normal ordering constant equal to $\frac{1}{2}$ and 0 for the antiperiodic (NS) and periodic (R) sectors of the NSR fermions.

For specific values of R_I and for specific choices of the background fields the $U(1)^6$ of the compactified torus is enlarged. To reproduce the $SO(12) \times E_8 \times E_8$ model of the previous section, the radius of the six compactified dimensions is taken at $R_I = \sqrt{2}$. The basis vectors e_i^I are the simple roots of $SO(12)$. The metric G_{ij} is the Cartan matrix of $SO(12)$ and the antisymmetric tensor field is given by,

$$B_{ij} = \begin{cases} G_{ij} & ; i > j, \\ 0 & ; i = j, \\ -G_{ij} & ; i < j. \end{cases} \tag{8}$$

The right–moving momenta produce the root vectors of $SO(12)$. For $R_I = \sqrt{2}$ and with the chosen background fields all the root vectors are massless, thus reproducing the same gauge group as in the free fermionic formulation.

The orbifold model is obtained by moding out the six dimensional torus by a discrete symmetry group, P. The allowed discrete symmetry groups are constrained by modular invariance. The Hilbert space is obtained by acting on the vacuum with twisted and untwisted oscillators and by projecting on states that are invariant

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under the space and group twists. A general left–right symmetric twist is given by $(\theta^i_j, v^i; \Theta^I_J, V^I)$ $(i = 1, \cdots, 6)$ $(I = 1, \cdots, 16)$ and $X^i(2\pi) = \theta^i_j X^j(0) + v^i$; $X^I(2\pi) = \Theta^I_J X^J(0) + V^I$. The massless spectrum contains mass states from the untwisted and twisted sectors. The untwisted sector is obtained by projecting on states that are invariant under the space and group twists. The twisted string centers around the points that are left fixed by the space twist. In the case of "standard embedding" one acts on the gauge degrees of freedom in an $SU(3) \in E_8 \times E_8$ with the same action as on the six compactified dimensions + NSR fermions. In this case the number of chiral families (27's of E_6) is given by one half the Euler characteristic,

$$\chi = \frac{1}{|P|} \sum_{g,h \in P} \chi(g, h),$$

where $\chi(g, h)$ is the number of points left fixed simultaneously by h and g. The mass formula for the right–movers in the twisted sectors is given by,

$$M_R^2 = -1 + \frac{(P + V)^2}{2} + \Delta c_\theta + N_R$$

where V^I are the shifts on the gauge sector and $\Delta c_\theta = \frac{1}{4} \sum_k \eta_k (1 - \eta_k)$ is the contribution of the twisted bosonic oscillators to the zero point energy and $\eta_k = \frac{1}{2}$ for a Z_2 twist.

To translate the fermionic boundary conditions to twists and shifts in the bosonic formulation we bosonize the real fermionic degrees of freedom, $\{y, \omega | \bar{y}, \bar{\omega}\}$. Defining, $\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = -ie^{iX_i}$, $\eta_i = \sqrt{\frac{1}{2}}(y_i - i\omega_i) = -ie^{-iX_i}$ with similar definitions for the right movers $\{\bar{y}, \bar{\omega}\}$ and $X^I(z, \bar{z}) = X^I_L(z) + X^I_R(\bar{z})$. With these definitions the world–sheet supercurrents in the bosonic and fermionic formulations are equivalent,

$$T_F^{int} = \sum_i \chi_i y_i \omega_i = i \sum_i \chi_i \xi_i \eta_i = \sum_i \chi_i \partial X_i.$$

The momenta P^I of the compactified scalars in the bosonic formulation are identical with the $U(1)$ charges $Q(f)$ of the unbroken Cartan generators of the four

dimensional gauge group,

$$Q(f) = \frac{1}{2}\alpha(f) + F(f)$$

where $\alpha(f)$ are the boundary conditions of complex fermions f, reduced to the interval $(-1, 1]$ and $F(f)$ is a fermion number operator.

The boundary condition vectors b_1 and b_2 now translate into $Z_2 \times Z_2$ twists on the bosons X_i and fermions χ_i and to shifts on the gauge degrees of freedom. The massless spectrum of the resulting orbifold model consist of the untwisted sector and three twisted sectors, θ, θ' and $\theta\theta'$. From the untwisted sector we obtain the generators of the $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ gauge groups. The only roots of $SO(12)$ that are invariant under the $Z_2 \times Z_2$ twist are those of the subgroup $SO(4)^3$. Thus, the $SO(12)$ symmetry is broken to $SO(4)^3$. Similarly, the shift in the gauge sector breaks one E_8 symmetry to $E_6 \times U(1)^2$. In addition to the gauge group generators the untwisted sector produces: three copies of $27 + \bar{27}$, one pair for each of the complexified NSR left–moving fermions; three copies of, $1 + \bar{1}$, E_6 singlets which are charged under $U(1)^2$. The $E_8 \times E_8$ singlets are obtained from the root lattice of $SO(12)$ and transform as $(1, 4, 4)$ under the $S0(4)^3$ symmetries, one for each of the complexified NSR left–moving fermions.

The number of fixed points in each twist is 32. The total number of fixed points is 48. The number of chiral 27's is 24, eight from each twisted sector, and matches the number of chiral 27's in the fermionic model. For every fixed point we obtain the $SO(4)^3 \times E_6 \times E_8$ singlets. These are obtained for appropriate choices of the momentum vectors, P^I. The $E_6 \times E_8$ singlets can be obtained by acting on the vacuum with twisted oscillators and from combinations of the dual of the invariant lattice, I^*, [16]. The spectrum of the orbifold model and its symmetries are seen to coincide with the spectrum and symmetries of the fermionic model.

4. The realistic free fermionic models

The previous results suggest that there is a correspondence between the models in the fermionic and orbifold formulations. The important point to realize is that in the fermionic formulation the 12 internal fermionic states, $\{y, w | \bar{y}, \bar{\omega}\}$, correspond to the six dimensional "compactified space" of the orbifold. The 16 complex fermionic states $\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\cdots,8}\}$ correspond to the gauge sector, and $\chi^{1,\cdots,6}$ correspond to the RNS fermions, of the orbifold model. The boundary conditions, assigned to the internal fermions $\{y, w | \bar{y}, \bar{\omega}\}$, determine many of the properties of the low energy spectrum. The number of generations, the presence of Higgs doublets and the projection of Higgs triplets, the allowed cubic and quartic order terms in the superpotential are shown to be determined by the specific assignment of boundary conditions to these set of internal fermions. Thus, in the realistic free fermionic models, we learn how the internal space determines the low energy properties of the standard model, without an exact knowledge of what is the action (of the additional "Wilson line") on the internal orbifold.

In the realistic free fermionic models the boundary condition vector ξ_1 is replaced by the vector 2γ in which $\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\}$ are periodic and the remaining left– and right–moving fermionic states are antiperiodic. The set $\{1, S, 2\gamma, \xi_2\}$ generates a model with $N = 4$ space–time supersymmetry and $SO(12) \times SO(16) \times SO(16)$ gauge group. The b_1 and b_2 twist are applied to reduce the number of supersymmetries from $N = 4$ to $N = 1$ space–time supersymmetry. The gauge group is broken to $SO(4)^3 \times U(1)^3 \times SO(10) \times E_8$. The $U(1)$ combination $U(1) = U(1)_1 + U(1)_2 + U(1)_3$ has a non–vanishing trace and the trace of the two orthogonal combinations vanishes. The number of generations is still 24 with a combinatorial factor for each sector b_1, b_2 and b_3 as in Eq. (3). The chiral generations are now 16 of $SO(10)$ from the sectors b_j $(j = 1, 2, 3)$. The $10 + 1$ and the E_6 singlets from the sectors $b_j + \xi_1$ are replaced by vectorial 16 of the hidden $SO(16)$ gauge group from the sectors $b_j + 2\gamma$.

The realistic free fermionic models are obtained by moding out the symmetry

with three additional boundary condition vectors that correspond to Wilson lines in the orbifold formulation. The number of generations is reduced to three generations, one from each twisted sector b_1, b_2 and b_3 by reducing the combinatorial factor in Eq. (3) from eight to one. Each additional vector acts simultaneously on each complex plane as a Z_2 twist, thus reducing the number of generations to exactly one generation from each sector b_1, b_2 and b_3. Each chiral generation is obtained from a distinct twisted sector of the orbifold model and none from the untwisted sector. the reduction to three generations is correlated with the breaking of the $SO(4)^3$ horizontal symmetries to factors of $U(1)'s$ [13]. This is however possible due to the fact that we started from a $Z_2 \times Z_2$ orbifold with the specific choice of radii and background fields, thus producing the degeneracy of zero modes as in Eq. (3), or alternatively, producing exactly sixteen fixed points, or eight generations, in each twisted sector.

The underlying $Z_2 \times Z_2$ orbifold compactification has an important implication for quark and lepton flavor mixing. After applying the "Wilson line" projections each sector b_1, b_2 and b_3 produces one generation. The $SO(10)$ symmetry is broken to a subgroup that contains the standard model gauge group [7,9] or is exactly the standard model gauge group times a $U(1)$ [8,11]. The light Higgs doublets are obtained from the Neveu–Schwarz sector and from a combination of the additional "Wilson line", and transform as the vector representation of $SO(10)$. The standard model gauge group and its matter content have the traditional $SO(10)$ embedding. Thus, the weak hypercharge is well defined and unambiguous. The fermion mass terms in the low energy effective superpotential are obtained from renormalizable and nonrenormalizable terms that are invariant under all the symmetries of the string models [14]. The nonrenormalizable terms become effective renormalizable terms after giving non–vanishing VEVs to some scalar singlets in the massless spectrum of the string models. The effective renormalizable terms are suppressed relative to the terms that are obtained directly at the cubic level. In this manner one obtains hierarchical fermion mass and mixing terms [15]. The sector b_3 produces the lightest generation states while one of b_1 and b_2 produces the heavy

generation states and the other produces the second generation states [14]. The nonrenormalizable terms that mix between the generations have a generic form

$$f_i f_j h V_i \bar{V}_j \frac{\phi^n}{M^{n+2}},$$

where f_i and f_j are fermion states from the sectors b_i, b_j with $i \neq j$, h represent the two light Higgs representations, V_i and $\bar{V}_j$ are two scalars from the sectors $b_i + 2\gamma$ and $b_j + 2\gamma$, ϕ^n is a combination of scalar $SO(10) \times SO(16)$ singlets and $M \sim 10^{18} GeV$ [15,12]. If the states from the sectors $b_j + 2\gamma$ get non–vanishing VEVs of order $O(\frac{1}{10}M)$ semi–realistic quark mixing matrices can be obtained in these models [12]. We observe that the generic texture of these terms is a result of the underlying $Z_2 \times Z_2$ orbifold compactification. Namely, the texture of the mixing terms is of the generic form $16_i 16_j 1016_i 16_j \phi^n$, where the first two 16 are in the spinorial representation of the observable $SO(10)$, the 10 is in the vector representation of the observable $SO10$), the last two 16 are in the vector representation of the hidden $SO(16)$ and ϕ^n is a combination of $SO(10) \times SO(16)$ scalar singlets.

In this paper I discussed the orbifold models that correspond to the realistic models in the free fermionic formulation. I illustrated in a specific example how the $Z_2 \times Z_2$ orbifold model with a specific choice of background fields and compactification radii reproduces the spectrum and symmetries of the free fermionic model. I suggest that the structure of the $Z_2 \times Z_2$ orbifold compactification with standard embedding and at the specific point in moduli space are the origin of the realistic features of free fermionic models. In particular, the "naturalness" of three generations, advocated in ref. [13] is seen to be a result of the $Z_2 \times Z_2$ orbifold compactification with standard embedding and at the point in compactification space that correspond to the free fermionic formulation. The free fermionic formulation correspond to toroidal compactification at the most symmetric point in compactification space. The $Z_2 \times Z_2$ orbifold is the most symmetric orbifold that one can construct at this point which is consistent with $N = 1$ space–time supersymmetry. We are intrigued by the fact that the most realistic string models to

date are constructed at the most symmetric point in compactification space. Could this be an accident? A better understanding of the correspondence between the realistic free fermionic models and other string formulation will hopefully provide further insight into the realistic features of free fermionic models.

Acknowledgments

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